QCD Pomeron with nonzero conformal spin from AdS/CFT Quantum Spectral Curve

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Abstract

In the present work we generalized the method of the AdS/CFT Quantum Spectral Curve (QSC) applied to the spectrum of N=4 supersymmetric Yang-Mills theory to the case of the both spins of the state being non-integer. Starting from the Q-system equations and the so-called gluing conditions following from the analytic properties of the Q-functions we reproduced the known results from a new point of view together with some new numerical and analytical ones. Namely, we rederived the Faddeev-Korchemsky Baxter equation for Lipatov spin chain with nonzero conformal spin reproducing the corresponding LO Balitsky-Fadin-Kuraev-Lipatov (BFKL) kernel eigenvalue. Also, we obtained the formula of the NNLO BFKL intercept function for arbitrary conformal spin and the numerical result for the twist-2 operator trajectory. The non-perturbative results include the slope-to-intercept function and the curvature functions in the BPS point for conformal spin n=1. All these results represent a non-trivial test of the QSC method.

Introduction

In this work we consider the generalisation of the BFKL regime adding a nonzero conformal spin. As we restrict our consideration to the class of the twist-2 $\mathfrak{sl}(2)$ operators of the form

$$\mathcal{O} = \mathrm{tr} Z D_+^S Z + (\text{permutations}),$$

which were analyzed by the QSC methods in [1, 2], addition of the non-zero conformal spin is expressed by

$$\mathcal{O} = \mathrm{tr} Z D_+^{S_1} \partial_\perp^{S_2} Z + (\text{permutations}).$$

As we know from the BFKL calculations, the first spin S_1 is identified with the pomeron spin S and the second spin S_2 is identical to the conformal spin n in the high energy scattering framework.

Motivation

- Using the methods of the Quantum Spectral Curve (QSC) [1, 3] originating from integrability of $\mathcal{N}=4$ Super–Yang-Mills theory analytically continue the scaling dimensions of twist-2 operators and reproduce the so-called pomeron eigenvalue of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation with nonzero conformal spin [4].
- Derive the Faddeev-Korchemsky Baxter equation with nonzero conformal spin for the Lipatov's spin chain known from the integrability of the gauge theory in the BFKL limit.
- Find a way for systematic expansion in the scaling parameter in the BFKL regime and study the Pomeron trajectory by numerical and analytical algorithms of QSC.

Gluing conditions for Quantum Spectral Curve

Analytical structure

Analytical structure of the **P**- and **Q**-functions on their defining sheet of the Riemann surface

$$\mathbf{P}_a(u) = x^{-\tilde{M}_a}(u)\left(g^{-\tilde{M}_a}A_a + \sum_{k=1}^{+\infty} c_{a,k}/x^{2k}(u)\right),$$

$$\mathbf{Q}_i(u) = B_i u^{\hat{M}_i - 1}\left(1 + \sum_{k=1}^{+\infty} B_{i,k}/u^k\right).$$

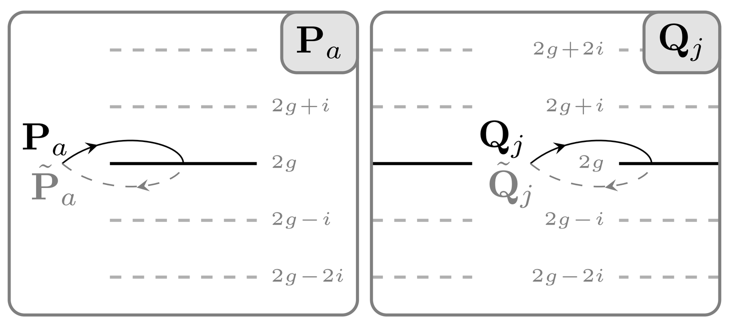

Figure 1: Analytic properties of **P**- and **Q**-functions.

The equation for the UHPA $Q_{a|i}$ functions

$$Q_{a|i}^+ - Q_{a|i}^- = \mathbf{P}_a \mathbf{Q}_i, \quad \mathbf{P}^a = \left(Q^{a|i}\right)^+ \mathbf{Q}_i, \quad \mathbf{Q}^i = \left(Q^{a|i}\right)^+ \mathbf{P}_a.$$

Algebraic structure

- Now we are to formulate the general form of the gluing conditions. We have two matrices connecting the Q functions on the different sheets of the Riemann surface [5]

$$\tilde{\mathbf{Q}}^i(u) = M^{ij}(u)\bar{\mathbf{Q}}_j(u),$$
$$\tilde{\mathbf{Q}}^i(u) = L^{ij}(u)\mathbf{Q}_j(-u),$$

- The matrices M and L satisfy

$$\bar{M}^t(u) = M(u), \quad L^t(-u) = L(u).$$

Integer and non-integer spins

- We have the restrictions on the gluing matrix

$$M^{ij}(-u)\bar{\mathbf{Q}}_j(-u) = L^{ij}(-u)\tilde{\mathbf{Q}}_j(u),$$
$$M^{ik}\Omega_k^j = -M^{jk}\Omega_k^i.$$

The obtained equations are valid for any values of the coupling g and spins S_1 and S_2. The obtained solutions allow us to solve the QSC numerically.

- For the integer spins S_1 and S_2 we obtain

$$M = \begin{pmatrix} 0 & i\alpha & 0 & 0 \\ -i\alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & \pm\frac{i}{\alpha} \\ 0 & 0 & \mp\frac{i}{\alpha} & 0 \end{pmatrix}$$

- For non-integer spins S_1 and S_2 we have

$$M = \begin{pmatrix} c_1 & c_2 & c_{31} & c_{41} \\ b_1 & 0 & 0 & 0 \\ d_{31} & 0 & d_1 & d_2 \\ a_{41} & 0 & a_1 & a_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 & c_{32} & c_{42} \\ 0 & 0 & 0 & 0 \\ d_{32} & 0 & 0 & 0 \\ d_{42} & 0 & 0 & 0 \end{pmatrix} e^{2\pi u} + \begin{pmatrix} 0 & 0 & c_{33} & c_{43} \\ 0 & 0 & 0 & 0 \\ d_{33} & 0 & 0 & 0 \\ d_{43} & 0 & 0 & 0 \end{pmatrix} e^{-2\pi u}.$$

Numerical results for conformal spin $n = 1$

Using the method of Quantum Spectral Curve and asymptotics of the Q-functions described above we are able to numerically calculate [5] the following quantities.

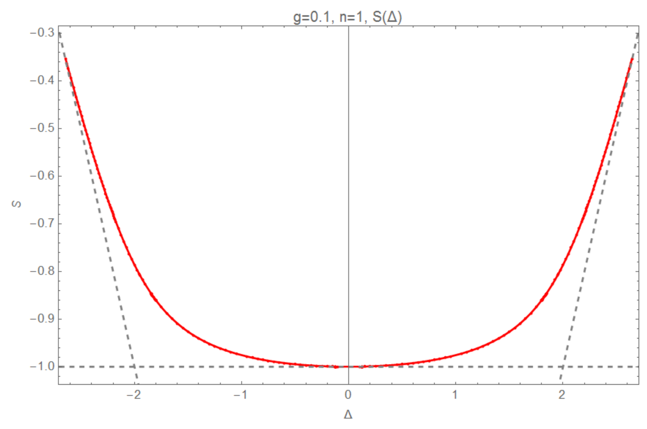

Figure 2: Trajectory of $S(\Delta)$ for $g = 1/10$.

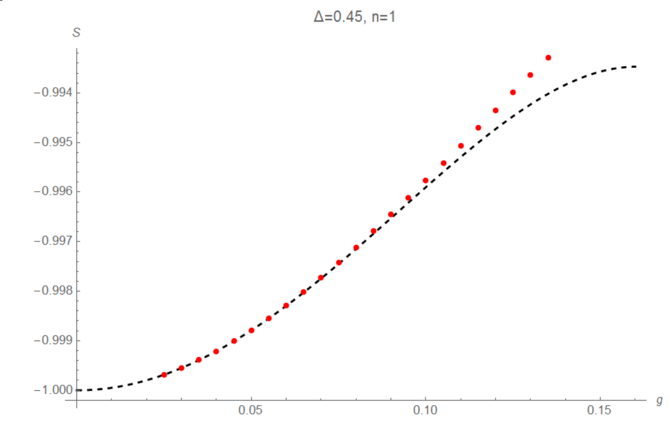

Figure 3: Dependence of S on g for fixed $\Delta = 0.45$.

	Fit of numerics	Exact perturbative
LO	0.509195398361183370691859	0.509195398361183370691860
NLO	-9.9263626361061612225	-9.9263626361061612225
NNLO	151.9290181554014	?
NNNLO	-2136.77907308	?

Table 1: Numerical fitting of the BFKL eigenvalues in the first four orders for $\Delta = 0.45$.

Weak coupling expansion

The LO, NLO and NNLO intercepts are

$$j_{LO} = 4\mathbb{S}_1$$
$$j_{NLO} = 2(\mathbb{S}_{2,1} + \mathbb{S}_3) + \frac{\pi^2}{3}\mathbb{S}_1,$$
$$j_{NNLO} = 32(\mathbb{S}_{1,4} - \mathbb{S}_{3,2} - \mathbb{S}_{1,2,2} - \mathbb{S}_{2,2,1} - 2\mathbb{S}_{2,3}) - \frac{16\pi^2}{3}\mathbb{S}_3 - \frac{32\pi^2}{45}\mathbb{S}_1,$$

where $\mathbb{S}$ is a binomial harmonic sum. The latter is in the complete agreement with the results of [6].

Near BPS all-loop expansion

Slope-to-intercept function

$$\theta(g) = \left.\frac{dS}{dn}\right|_{\substack{\Delta=0\\n=1}} = 1 + \frac{I_1(4\pi g)I_2(4\pi g)}{\sum_{k=1}^{+\infty}(-1)^k I_k(4\pi g)I_{k+1}(4\pi g)}.$$

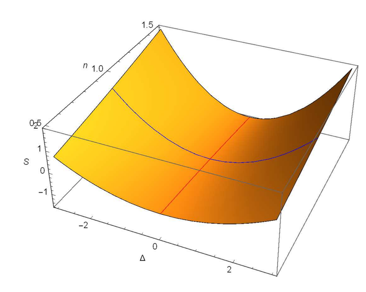

Figure 4: Trajectory $S(\Delta, n)$.

Curvature function

$$\gamma(g) = \left.\frac{d^2S}{d\Delta^2}\right|_{\substack{\Delta=0\\n=1}} =$$

$$= \frac{1}{4\pi g^4 I_2^2}\oint_{-2g}^{2g} dv\left(\cosh_-^{v}\, v\Gamma[\cosh_-^{u}\, u](v) - \cosh_-^{v}\, v^2\Gamma[\cosh_-^{u}](v)\right) +$$

$$+ \frac{1}{16\pi g^5 I_2}\oint_{-2g}^{2g} dv\left(\frac{v^3\Gamma[\cosh_-^{u}](v) - 2v^2\Gamma[\cosh_-^{u}\, u](v) + v\Gamma[\cosh_-^{u}\, u^2]}{x_v - \frac{1}{x_v}}\right)$$

where $\Gamma[h(v)](u) = \oint_{-2g}^{2g}\frac{dv}{2\pi i}\partial_u \log\frac{\Gamma[i(u-v)+1]}{\Gamma[-i(u-v)+1]}h(v)$.

Conclusions and outlook

- In our work we managed to reproduce the dimension of twist-2 operator with conformal spin of $\mathcal{N} = 4$ SYM theory in the 't Hooft limit in the leading order (LO) of the BFKL regime directly from exact equations for the spectrum of local operators called the Quantum Spectral Curve.
- We managed to find two nonperturbative quantities and this is one of a very few examples of all-loop calculations, with all wrapping corrections included, where the integrability result can be checked by direct Feynman graph summation of the original BFKL approach.
- Using the iterative procedure, there was obtained the BFKL intercept for arbitrary conformal spin up to NNLO order in terms of the binomial harmonic sums.
- By application of the QSC numerical algorithm there were calculated the operator trajectories $S(\Delta, n)$ for different values of the conformal spin n and coupling constant g.
- The ultimate goal of the BFKL approximation to QSC would be to find an algorithmic way of generation of any BFKL correction (NNLO [5], NNNLO, etc) on Mathematica program, similarly to the one for the weak coupling expansion via QSC.

References

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