Optimal statistical decision for Gaussian graphical model selection.

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Gaussian graphical model is a graph representation of dependence structure for Gaussian random vector (Edwards, D.2000, Lauritzen S.L.1996). It is recognized as powerful tool in different applied fields such as bioinformatics, error-control codes, speech language and information retrieval and others (Jordan M.I, 2004). Gaussian graphical model selection is statistical problem to identify Gaussian graphical model from sample of given size.

Different approaches for Gaussian graphical model selection are suggested in the literature. One of them is based on considering the family of individual conditional independence tests (Dempster A P, 1972). Application of this approach leads to construction of variety of multiple testing statistical procedures for Gaussian graphical model selection (Drton M. Perlman M., 2007). Important characteristic for these procedures is its error rate for given sample size.

In existing literature the great attention is paid to control of error rates for incorrect edge inclusion (Type I error). However in graphical model selection it is important to take into account error rates for incorrect edge exclusion too (Type II error). To handle this issue we consider graphical model selection problem in the framework of multiple decision theory. Quality of statistical procedures is measured by risk function with additive losses (Lehmann E.L., 1957). Additive losses allow to take into account both types of errors. We construct optimal unbiased tests of Neyman structure (Koldanov et al., 2017) for individual hypotheses and combine it to obtain a multiple decision statistical procedure. We show that obtained procedure is optimal in the sense that it minimizes linear combination of expected numbers of Type I and Type II errors in the class of unbiased multiple decision procedures. Detailed results of the talk are given in ([Kalyagin](https://arxiv.org/find/stat/1/au%3A%2BKalyagin_V/0/1/0/all/0/1) et al. 2017).

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