

## Macromodeling of linear equivalent electrical circuits.

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**Abstract.** The essence of the work lies in the formal transformation of models of linear or linearized equivalent electrical circuit, formed by using methods of artificial electrorloge, in the macromodel, which with the same precision, but with an increase of several orders of magnitude speed, you can calculate the same output characteristics as the model. Source models can serve not only the equivalent electrical circuit, but the linear model obtained by methods of finite elements or finite differences in various subject areas, where there is a streaming potential and phase variables. These areas can be mechanics, electrodynamics, thermal fields, acoustics, etc.

**Keywords:** Linear equivalent; Artificial electrorloge; Macromodel; Method of A. M. Lyapunov; Irregular polynomial matrix; Gauss-Jordan alpha numeric matrix; Sparse matrix; Polynomial matrix; Large-scale problems; Reduce; Parametric optimization.

The essence of the work lies in the formal transformation of models of linear or linearized equivalent electrical circuit, formed by using methods of artificial electrorloge, in the macromodel, which with the same precision, but with an increase of several orders of magnitude speed, you can calculate the same output characteristics as the model.

These features include:

- static characteristics;
- frequency characteristics;
- the zeros and poles of system functions;
- dynamic features (in the form of one or several analytic expressions and in numerical forms);
- eigenvalues and vectors of a matrix macro model (irregular polynomial to a high degree and relatively small order) to determine the stability and margin of stability of matrix the original scheme on the first method of A. M. Lyapunov, its self-resonant frequency and duration of the transition process;
- partial derivatives listed above of the output characteristics at a small number of variable parameters of the scheme for the replacement of optimization methods 1st order optimized in macromodel.

Macromodel can be used:

- for replacement of the respective subcircuits of the macro models (methods of formation models consisting of macromodels developed) that will allow you to effectively analyze large-scale problems;
- to create effective new element basis of design (if the original scheme formed according to standard designs);
- to carry out parametric optimization for the corresponding macro model.

For explanation of work to a first approximation can be used the following mathematical expression.

If the model of the initial scheme formed in the extended homogeneous coordinate basis for the analysis in the frequency domain, has the form

$$\begin{bmatrix} C_{11}p + G_{11} & \vdots & C_{12}p + G_{12} \\ \dots & \dots & \dots \\ C_{21}p + G_{21} & \vdots & C_{22}(\bar{Q})p + G_{22}(\bar{Q}) \end{bmatrix} \begin{bmatrix} \bar{X}_1 \\ \dots \\ \bar{X}_2 \end{bmatrix} = \begin{bmatrix} \bar{Y}_1 \\ \dots \\ \bar{Y}_2 \end{bmatrix}, \quad (1)$$

где  $C_{11}p + G_{11}$  -  $(M \times M)$ - matrix,  $C_{22}(\bar{Q})p + G_{22}(\bar{Q})$  -  $(m \times m)$  - matrix,  $m \ll M$ ,  $\bar{Q}$ , is a vector of variable parameters,  $\bar{X}_2$  - a vector of exogenous variables that includes the ratio of the "input/output", simple reliance on a small number of variable parameters, the macromodel circuit (1) can be written in the following form

$$\begin{aligned} [-(C_{21}p + G_{21})(C_{11}p + G_{11})^{-1}(C_{12}p + G_{12}) + C_{22}(\bar{Q})p + G_{22}(\bar{Q})]\bar{X}_2 = \\ = \bar{Y}_2 - (C_{21}p + G_{21})(C_{11}p + G_{11})^{-1}\bar{Y}_1. \end{aligned} \quad (2)$$

From (2) we see that the basis of building a macro model is the analytical form of the matrix  $C_{11}p + G_{11}$ . This solves a generalized problem of the eigenvalues and eigenvectors of this irregular polynomial matrix 1-th degree with a mandatory calculating the zero and infinite eigenvalues that need to build a macromodel. This assumes that the geometric multiplicity of the eigenvalues does not exceed the algebraic (which is practically guaranteed for approximately 100% of the task).

From these relations it becomes clear that as the size of memory required to store macro model and the size of the problem being solved are reduced to  $M/m$  times. In the cubic according to the complexity analysis of the size of the problem being solved becomes clear the effectiveness of the proposed methods. In addition to frequency analysis based on special transformations, Gauss-Jordan alpha numeric matrix (letters are variable parameters), scaling tasks for functions of 2 variables, additionally, the complexity of tasks is reduced in  $8M$  times.

Until now it was assumed that the matrix  $C_{11}p + G_{11}$  is dense. However, it can be sparse or have special structure. Both of these factors were taken into account when constructing macro model. Note that the complexity of the calculations  $(C_{11}p + G_{11})^{-1}$  in analytical form for dense matrices is  $M^4$  approximately real multiplicative operations.

For sparse matrix  $C_{11}p + G_{11}$  has been developed a method of constructing a macromodel based on its defining values. As a result, the complexity of building a macro model has been reduced about in  $M^2$  times. It is clear that about  $M=1000$ , the complexity is reduced by about 1000000 times.

For a block-diagonal matrix with a double-edging (which is typical for large electrical circuits) was developed a method of hierarchical macro-modeling based on the appeal to the analytical form of polynomial matrices of high degrees. In particular, such a matrix can serve as the matrix  $-(C_{21}p + G_{21})(C_{11}p + G_{11})^{-1}(C_{12}p + G_{12}) + C_{22}(\bar{Q})p + G_{22}(\bar{Q})$  with loss depends on varied parameters. Anyway, regardless of how formed, the circuit consisting of the macromodels, the possible implementation of the hierarchical macro-modeling of linear circuits.

Examples of usage.

1. Ministry of science of the Russian Federation. Model schemes of 50 equations. The optimization model in the frequency region around 4 hours. Build time macro model – 15 minutes. Time optimising macro model  $\frac{1}{2}$  seconds. In both cases, the method of the optimization served as the method of Fletcher-Reeves with 8 variable parameters.

2. The satellite mobile communication (recent years). Model – 500000 ordinary differential equations. 3 distributed in all equations the variable parameter. Macromodel – 400 equations explicitly included in the variable parameters. Build time macro model is equal to the time analysis of the model at one point in time.

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